Early High School Mathematics — Performance Level Descriptors

Reporting Categories	Needs Support	Close	Ready	Exceeding
Number and Quantity Focus is on rational exponents and developing a richer understanding of quantity including derived units.	*A student performing at the Needs Support level:* • finds rational approximations of irrational numbers, perhaps using a calculator.	*A student performing at the Close level:* • uses rational approximations of irrational numbers to compare and order rational and irrational numbers.	*A student performing at the Ready level:* • Rewrites the square root of a whole number as $k\sqrt{p}$, where k and p are whole numbers and p is as small as possible. • adds or subtracts multiples of the same radical. • multiplies or divides radicals by rational numbers. • rewrites and evaluates expressions that involve rational exponents. • uses units to understand and manipulate the quantities used in solving a multi-step problem. • Classifies square roots as rational or irrational using the fact that $\sqrt{p}$ is rational whenever p is a perfect square and irrational whenever p is any other whole number.	*A student performing at the Exceeding level:* • evaluates algebraic expressions involving square roots. • rewrites numerical radical expressions in equivalent radical forms. • understands that measurements involve some degree of imprecision and reports results appropriately given the limitations and the context of the measurement.
Algebra Focus is on understanding structure of expressions and equations. Students use operations strategically to transform expressions and solve problems, connecting algebraic and graphical solutions.	*A student performing at the Needs Support level:* • rewrites a linear equation to put it into slope-intercept form. • performs operations on monomials of degree two or higher. • creates linear equations in one variable to represent a contextual situation. • determines whether a given number makes an absolute value inequality true. • determines whether a given ordered pair is a solution to a system of equations. • rewrites monomial expressions raised to a whole-number power using properties of exponents. • evaluates square roots of perfect squares. • evaluates cube roots of perfect cubes. • converts numbers to and from scientific notation. • uses algebraic properties and operations strategically to rewrite numeric expressions. Example: Given (4)(19)(25) and seeing from the structure that multiplying (4)(25) first will make the calculation easier.	*A student performing at the Close level:* • interprets the meaning of parameters or variables used in a problem in terms of the context. • rewrites rational expressions involving monomials in both the numerator and denominator into forms with positive exponents. • identifies zeros of polynomials given a factorization into linear factors. • understands that $\sqrt{p}$ is a solution to $x^2 = p$ for nonnegative p. • understands that $\sqrt[3]{p}$ is a solution to $x^3 = p$. • creates a quadratic equation to represent a contextual situation involving multiplication of quantities or given an appropriate formula. • creates the factored form of a quadratic that has two given zeros. • determines whether a single-variable compound inequality true for a given number. • rearranges a formula to highlight a specific quantity of interest when that quantity is represented by a single variable, the formula is linear in that variable, and the variable appears only once in the formula. • approximates the solution to a system of linear equations by graphing, and verifies the solution. • graphs the solution set of a linear inequality. • solves a single-variable linear inequality and graphs the solution set. • solves a system of two equations in two variables with the use of technology or provided graphs. • understands that a system of linear equations that represents distinct parallel lines has no solution. • understands that a system of linear equations where each equation represents the same line has infinitely many solutions. • solves one-variable linear equations. • graphs linear equations including systems of linear equations. • uses algebraic properties and operations strategically to rewrite monomials. Example: Given $(xy^2)(xy^2)(xy^2)$ and recognizing this as a cube.	*A student performing at the Ready level:* • analyzes the structure of an expression and identifies ways to rewrite it. • sees a complicated expression as a combination of simpler expressions and interprets the expressions in terms of the quantities that they represent in a context. • finds the sum or difference of two polynomials. • multiplies binomials using the distributive property. • factors quadratic expressions. • rewrites rational expressions involving monomials in both the numerator and denominator into a single term with positive and negative exponents. • produces a polynomial with a given list of zeros. • creates a system of linear equations to represent a contextual situation. • understands that $x = \sqrt{p}$ and $x = -\sqrt{p}$ represent all the solutions to $x^2 = p$, for nonnegative p. • creates a quadratic equation from straightforward information about zeros, other points on the graph, coefficients, or the vertex. • rearranges a formula to highlight a specific quantity of interest. • solves and graphs the solution set of an inequality equivalent to $\lvert x\rvert < c$, $\lvert x\rvert > c$, $\lvert x\rvert \le c$, or $\lvert x\rvert \ge c$. • solves a system of linear equations. • graphs the solution set of a system of linear inequalities. • understands that a system of two linear equations representing lines that are not parallel always has exactly one solution. • finds one or two solutions to a system of two linear equations that each represent the same line. • understands that adding a real number to both sides of an equation, or multiplying both sides of an equation by a nonzero real number, does not change the solutions, and understands that property supports a method for solving all linear equations. • performs operations on numbers written in scientific notation. • uses algebraic properties and operations strategically to rewrite polynomials. Example: Given $(x-4)(2x+3)(x+4)$ and seeing from the structure that multiplying $(x-4)(x+4)$ first will make the calculation easier.	*A student performing at the Exceeding level:* • uses the structure of an expression to make deductions about the expression, such as when it is zero or what its minimum value is. • produces equivalent forms of an expression to reveal properties or quantities of interest as needed. • applies the Remainder Theorem to determine if a polynomial has a given factor. • rewrites rational expressions with polynomial numerator and denominator in equivalent forms, using inspection or long division. • multiplies trinomials using the distributive property. • produces two inequivalent polynomials, each with the same given list of zeros. • creates a simple rational or exponential equation to represent a contextual situation. • creates a quadratic equation to model the height of an object under the influence of gravity. • understands that absolute value equations can often be solved through the use of cases. • solves a system of two equations in two variables, where one equation is linear and one is quadratic. • understands that multiplying both sides of an equation by an expression can change the solutions, but only when the expression is zero or undefined. • understands that the only way two expressions multiplied together give a result of zero is when the value of at least one of the expressions is zero, and that this property can be used help solve some equations. • understands that squaring both sides of an equation keeps all of the original solutions but may add extraneous solutions. • solves problems in contextual situations with quantities written in scientific notation. • chooses an appropriate/efficient method when solving systems of linear equations, quadratic functions, simple rational equations, and when rewriting polynomials.
Functions Focus is on linear, quadratic, and exponential functions, comparing their properties and using them to solve problems.	*A student performing at the Needs Support level:* • identifies key features on graphs such as minimums, maximums, and intercepts. • constructs simple functions using function notation to explicitly express a relationship between two quantities. • recognizes situations that are well modeled with linear functions. • constructs a linear function from a given table of values. • constructs a linear function to model the relationship between two quantities and uses it to solve a problem. • uses linear functions to solve problems. • identifies whether the slope of a given line is positive or negative. • graphs linear equations, including those in systems, when the equations are given in slope-intercept form. • identifies key characteristics of a linear function from its graph.	*A student performing at the Close level:* • evaluates functions for a given input that is in the domain of the function. • uses function notation to recognize the name of the function and the function's arguments. • matches a quadratic equation to a graph. • finds the inverse of linear functions of the form $f(x) = kx$ or $g(x) = x + k$. • recognizes situations that are well modeled with exponential functions. • recognizes the graph of a linear function that represents a contextual situation. • calculates slope as the change of y over the change of x for a linear function. • identifies lines on a graph that appear to have slope that is zero or undefined. • examines claims and makes explicit use of definitions about functions.	*A student performing at the Ready level:* • calculates the average rate of change of a function over a specified interval. • identifies from the graph of a nonlinear function whether the average rate of change over a given interval is positive, negative, or approximately zero. • identifies the valid inputs (domain) of a function from a graph, context, or from the algebraic expression of the function as a polynomial, as the square root of a linear expression, or as a rational expression with a linear denominator. • identifies the possible outputs (range) of a function from its graph. • expands a geometric sequence using a formula for the *nth* term of the sequence to find a given term. • builds a new function by combining two or more functions with arithmetic operations. • finds the inverse of non-constant linear functions. • understands the relation between the graph of $f(x)$ and the graph of $f(x) + c$ in terms of transformations. • recognizes situations that are well modeled with quadratic functions. • determines whether a relation is a function given a graph, equation, or table of values. • understands that slope represents a constant rate of change. • states the meaning of $f(x)$, $y = mx + b$, and other symbols they choose, including using the equal sign consistently and appropriately. • is careful about specifying units of measurements and labeling axes to clarify the correspondence with quantities in a problem. • calculates accurately and efficiently, and expresses numerical answers with a degree of precision appropriate for the problem context.	*A student performing at the Exceeding level:* • understands the relationship between the zeros of a quadratic function and the x-intercepts of the function's graph. • recognizes a function graph that represents all of the features from a given contextual situation, described in terms such as increasing, decreasing, remaining constant, with given inputs and outputs, minimum and maximum, average rates of change on intervals, and relations between these descriptors. • identifies the valid outputs (range) of a quadratic function from its equation. • writes arithmetic and geometric sequences as explicit or recursive formulas and uses them to model situations. • understands the relation between the graph of $f(x)$ and the graphs of $f(-x)$, $f(ax)$, $af(x)$, and $f(x + b)$ in terms of transformations. • constructs an exponential or quadratic model from a graph, contextual situation, or table of values, recognizing which of these function families is the most appropriate choice. • understands the foundations of functions (e.g., domain and range, one on one, function notation, constant rate of change); is able to apply understanding accurately in context; and is able to communicate the understanding verbally or in writing.

Early High School Mathematics — Performance Level Descriptors (Continued)

Reporting Categories	Needs Support	Close	Ready	Exceeding
Geometry Focus is on geometric relationships, including relationships in the coordinate plane, distance, and transformations. Students gain understanding of volume formulas for geometric objects.	*A student performing at the Needs Support level:* • recognizes similar triangles by angle-angle similarity. • understands characteristics of a circle (e.g., center, radius, degrees, and constant distance from a point). • identifies the midpoint of a line segment in the coordinate plane by its coordinates. • recognizes that an equation in the form $x^2 + y^2 = b$, where $b > 0$, is represented by a circle in the coordinate plane. • evaluates volume of cylinders, cones, pyramids, and spheres given appropriate measurements, perhaps by identifying and using an appropriate formula. • identifies or sketches the image of a point that has been translated in the coordinate plane. • finds a scale factor given similar figures. • understands that corresponding angles of similar figures are congruent. • understands translation, reflection, rotation, and dilation as kinds of transformations that can be applied to geometric figures. • constructs arguments using concrete referents such as objects, drawings, diagrams, and actions.	*A student performing at the Close level:* • recognizes when geometric figures are congruent using congruency theorems. • recognizes the corresponding angles and sides of two similar triangles and uses correct notation to state that the two triangles are similar. • recognizes that all circles can be shown to be similar by using dilation and translation, and provides the dilation/translation necessary to take one to the other. • interprets the parameters of an equation for a circle in the form $x^2 + y^2 = r^2$ and identifies the center as (0,0) and the radius as r, where r is positive. • determines whether two lines are parallel or perpendicular when given the slopes of the lines. • understands that a prism with base of area b and height h has volume bh no matter the shape of the base. • determines the coordinates of the image for a point when it is rotated 180° about a given center point in the coordinate plane. • identifies or sketches the image of a figure when it is reflected over the x- or y-axis in the coordinate plane. • identifies or sketches the image of a point when it is rotated about the origin. • finds the measure of any angle formed by parallel lines cut by a transversal given the measure of other angles. • constructs arguments, states assumptions, and recognizes and uses counterexamples.	*A student performing at the Ready level:* • determines that two figures are congruent by describing a series of transformations that will take one figure onto the other. • performs basic geometric constructions. • uses similarity criteria and proportional reasoning to solve for an unknown side length in two similar triangles. • recognizes one polygon is a dilation of another if corresponding sides are proportional and corresponding angles are congruent. • identifies, describes, and uses commonly known relationships among inscribed angles, radii, and chords (e.g., inscribed angle measures, bisector of chord passes through center, intersecting chords divided into lengths with the same product). • graphs the equation of a circle given in the form $(x - h)^2 + (y - k)^2 = r^2$. • uses the distance formula, congruence criteria, and similarity criteria to decide whether two figures in the coordinate plane are similar, congruent, or neither. • finds the equation of a line through a given point and parallel or perpendicular to a given a line. • determines whether two lines are parallel, perpendicular, or neither given two points on each of the lines. • finds the volume of prisms and pyramids with bases that are not rectangles, triangles, or circles but that can be made from triangles and sectors of circles. • applies geometric concepts based on area and volume in modeling contextual situations. • uses the Pythagorean theorem to determine a missing side length in a right triangle. • makes and justifies conjectures about the relationship between the angles formed by parallel lines cut by a transversal. • uses the Pythagorean theorem to solve problems involving distances between points in the coordinate plane. • constructs simple arguments in the form of proofs, stating assumptions, definitions, and previously established results.	*A student performing at the Exceeding level:* • uses the definition of congruence or congruence theorems to support assertions about geometric figures. • proves theorems about similar triangles. • uses similarity criteria to define the relationships between the sides of two similar right triangles, leading to definitions of trigonometric ratios for acute angles. • finds arc lengths and areas of sectors of a circle given central angle measure and radius. • finds the endpoint of a line segment given its midpoint and the other endpoint. • uses geometric shapes, their measures, and their properties to describe objects and model contextual situations. • makes plausible arguments, recognizes correct logic and reasoning, and explains why something is a flaw.
Statistics and Probability Focus is on distributions, association, and sampling. Students pay attention to the context of the data and use various statistical displays to interpret data.	*A student performing at the Needs Support level:* • recognizes that a population parameter estimate based on sample data won't necessarily be exact. • uses data from an experiment to informally decide whether there is a significant difference between population parameters. • recognizes clear positive or negative association in data presented in a scatterplot. • creates stem-and-leaf plots and scatterplots for small datasets.	*A student performing at the Close level:* • compares empirical data to a model's predicted value to judge the reasonableness of the model. • understands that two events are independent if the occurrence of one does not affect the probability of the other. • identifies a situation that could reasonably represent the independent and dependent variables in a data set. • recognizes the scatterplot of a data set from a listing of the data or from a description of the data. • estimates a linear model given a scatterplot of data with an approximately linear relationship.	*A student performing at the Ready level:* • compares summary statistics given representations, perhaps in different forms, for two different sets of data. • recognizes the purpose of and difference between sample surveys, experimental studies, and observational studies. • critiques surveys, experiments, and observational studies to investigate research questions, identifying the population and sample, and keying in on possible bias. • determines whether two events are independent. • uses the multiplication rule to determine the joint probability of two independent events. • computes conditional probability from a two-way table. • creates and uses the equation of a linear model for data with an approximately linear relationship to solve problems in a contextual situation. • interprets a linear model of a scatterplot in a given contextual situation.	*A student performing at the Exceeding level:* • interprets differences in shape, center, and spread for two sets of data, including accounting for the possible effects of outliers and the context of the data. • assesses the fit of a model informally by plotting and analyzing residuals. • understands the purpose of both random sampling and randomly assigning to treatment groups in experimental studies. • describes compound events in terms of other events with "and," "or," and "not" and represents these events using diagrams.
Modeling Producing, interpreting, understanding, evaluating, and improving mathematical models.	*A student performing at the Needs Support level:* • recognizes linear equations that represent relationships presented in tables and graphs.	*A student performing at the Close level:* • recognizes linear equations that represent relationships presented in a context and interprets graphic representations of linear function models.	*A student performing at the Ready level:* • recognizes systems of linear equations that represent relationships presented in a context and interprets graphic representations of function models of linear systems.	*A student performing at the Exceeding level:* • uses graphs and diagrams to represent and interpret contextual situations, including linear systems and probability events. • recognizes quadratic equations that represent relationships presented in a context and interprets graphic representations of quadratic function models.
Justification and Explanation Giving reasons, explaining "Why?"	*A student performing at the Needs Support level:* • visually analyzes and restates the given information in preparation for presenting a proof. • explains steps of a procedure. • provides a counterexample. • uses a pattern or sequence to draw a conclusion. • utilizes and cites conditional statements, specific aspects of created visual representations, and/or computations or procedures to clarify an argument or draw a conclusion. • draws conclusions using both a specific and general evidentiary statement or provides general support for a claim in order to reach a conclusion.	*A student performing at the Close level:* • justifies and defends conclusions by explaining errors in reasoning or calculations, providing counterexamples, applying relevant classification schemes, and/or verifying statements or claims used to draw a conclusion. • examines claims and makes explicit use of definitions about functions.	*A student performing at the Ready level:* • provides a coherent, logical argument or solution pathway by providing evidence to support claims.	*A student performing at the Exceeding level:* • provides thorough justification and defends conclusions by using multiple, connected statements and incorporating justification techniques such as explaining errors in reasoning or calculations, providing counterexamples, applying relevant classification schemes, and/or verifying statements or claims used to draw a conclusion. • employs proof techniques such proof by cases and indirect proof.

ACT Aspire®

Early High School Mathematics — Performance Level Descriptors

(Continued)

Reporting Categories	Needs Support	Close	Ready	Exceeding
Integrating Essential Skills Integrate and continue to grow with topics from prior grades.	*A student performing at the Needs Support level:* • applies knowledge and skills from prior grades by solving problems including those requiring calculations with decimals and fractions, rewriting expressions with exponents, and using counting techniques. • understands definitions for simple geometry shapes in terms of their properties. • uses simple geometric theorems about angles. • understands the concepts of area and volume and explains them in terms of square and cubic units. • uses different graphical representations of data as needed. • determines the probability of events that can be represented by a uniform probability model. • draws inferences about a population based on the results of a random sample. • reasons quantitatively. • calculates measures of central tendency and graphs histograms.	*A student performing at the Close level:* • computes the probability of compound events using counting techniques, organized lists, tables, tree diagrams, or simulation results. • converts between measurement units and systems. • reasons quantitatively by making sense of quantities and their relationships in problem situations.	*A student performing at the Ready level:* • applies knowledge and skills from prior grades by operating with rational expressions. • interprets the spread of a distribution. • understands why random sampling is used for surveys. • reasons abstractly and quantitatively by making sense of quantities and their relationships in problem situations. • creates a coherent representation of the problem at hand and attends to the meaning of the quantities—not just how to compute them.	*A student performing at the Exceeding level:* • creates a coherent representation of the problem at hand and attends to the meaning of the quantities; not only how to compute them but also how to flexibly use different properties and methods to solve problems.
Mathematical Practices Collected PLDs that focus on mathematical practices.	*A student performing at the Needs Support level:* • constructs arguments using concrete referents such as objects, drawings, diagrams, and actions. • reasons quantitatively. • visually analyzes and restates the given information in preparation for presenting a proof. • explains steps of a procedure. • provides a counterexample. • uses a pattern or sequence to draw a conclusion. • utilizes and cites conditional statements, specific aspects of created visual representations, and/or computations or procedures to clarify an argument or draw a conclusion. • draws conclusions using both a specific and general evidentiary statement or provide general support for a claim in order to reach a conclusion. • recognizes linear equations that represent relationships presented in tables and graphs. • constructs arguments using concrete referents such as objects, drawings, diagrams, and actions. • uses algebraic properties and operations strategically to rewrite numeric expressions. Example: Given (4)(19)(25) and seeing from the structure that multiplying (4)(25) first will make the calculation easier.	*A student performing at the Close level:* • constructs arguments, state assumptions, and recognize and uses counter examples. • justifies and defends conclusions by explaining errors in reasoning or calculations, providing counterexamples, applying relevant classification schemes, and/or verifying statements or claims used to draw a conclusion. • recognizes linear equations that represent relationships presented in a context and interprets graphic representations of function models. • constructs arguments, state assumptions, and recognize and use counterexamples. • examines claims and makes explicit use of definitions. • uses algebraic properties and operations strategically to rewrite monomials. Example: Given $(xy^2)(xy^2)(xy^2)$ and recognizing this as a cube.	*A student performing at the Ready level:* • constructs arguments in the form of proofs to state assumptions, definitions, and previously established results. • applies knowledge and skills from prior grades by operating with rational expressions. • interprets the spread of a distribution. • understands why random sampling is used for surveys. • reasons abstractly and quantitatively by making sense of quantities and their relationships in problem situations. • creates a coherent representation of the problem at hand and attends to the meaning of the quantities not just how to compute them. • recognizes linear equations or systems of linear equations that represent relationships presented in a context and interprets graphic representations of function models. • constructs simple arguments in the form of proofs, stating assumptions, definitions, and previously established results. • states the meaning of $f(x)$, $y = mx + b$, and other symbols they choose, including using the equal sign consistently and appropriately. • is careful about specifying units of measurements and labeling axes to clarify the correspondence with quantities in a problem. • calculates accurately and efficiently, expresses numerical answers with a degree of precision appropriate for the problem context. • uses algebraic properties and operations strategically to rewrite polynomials. Example: Given $(x - 4)(2x + 3)(x + 4)$ and seeing from the structure that multiplying $(x - 4)(x + 4)$ first will make the calculation easier.	*A student performing at the Exceeding level:* • makes plausible arguments, recognizes correct logic and reasoning, identifies flaws in arguments and explains why they are flaws. • creates a coherent representation of the problem at hand and attends to the meaning of the quantities; not only how to compute them but also how to flexibly use different properties and methods to solve problems. • uses graphs and diagrams to represent and interpret contextual situations, including linear systems and probability events. • recognizes linear equations or systems of linear equations, or quadratic equations that represent relationships presented in a context and interprets graphic representations of function models. • has a thorough understanding of foundations of functions (domain and range, one on one, function notation, constant rate of change, etc.); is able to apply understanding accurately in context; and is able to communicate the understanding verbally or in writing. • solves problems in contextual situations with quantities written in scientific notation. • chooses an appropriate/efficient method when solving systems of linear equations, quadratic functions, simple rational equations, and when rewriting polynomials.